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Permutation Symmetry for Neutrino and Charged-Lepton Mass Matrices

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Abstract

The permutation symmetry S_3 is appplied to obtain two equal Majorana neutrino masses, while maintaining three different charged-lepton masses and suppressing neutrinoless double beta decay. The resulting radiative splitting of the two neutrinos is shown to be suitable for solar neutrino vacuum oscillations.

1 Introduction

There are three known charged leptons, e, μ, τ, with very different masses:

$$m_e = 0.511 \text{ MeV}, \quad m_\mu = 105.66 \text{ MeV}, \quad m_\tau = 1777 \text{ MeV}. \tag{1}$$

Their accompanying neutrinos, ν_e, ν_μ, ν_τ, are not necessarily mass eigenstates. In general,

$$\nu_\alpha = \sum_{i=1}^{3} U_{\alpha i} \nu_i, \tag{2}$$

where $\alpha = e, \mu, \tau$ and ν_i are mass eigenstates.

$$U = \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & -1/\sqrt{2} \\ -\sin\theta/\sqrt{2} & \cos\theta/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{3}$$

is a typical mixing matrix which allows one to understand the recent atmospheric neutrino data[1] and the long-standing solar neutrino deficit[2] in terms of neutrino oscillations. The form of U in Eq. (3) has been advocated by many authors[3]. It has the virtue of maximal mixing between ν_μ and ν_τ which agrees well with the atmospheric data, and it allows the solar data to be interpreted with either the small-angle or large-angle matter-enhanced neutrino-oscillation solution[4], or the necessarily large-angle vacuum solution.

The masses $m_{1,2,3}$ are now subject to the conditions that

$$\Delta m^2_{13} \simeq \Delta m^2_{23} \sim 10^{-3} \text{ eV}^2 \tag{4}$$

for atmospheric neutrino oscillations, and

$$\Delta m^2_{12} \sim 10^{-5} \ \text{ or } \ 10^{-10} \text{ eV}^2 \tag{5}$$

for solar matter-enhanced or vacuum neutrino oscillations, where $\Delta m^2_{ij} \equiv m^2_i - m^2_j$. This allows for the intriguing possibility that neutrino masses are degenerate[5] with very small splittings. However, since the charged-lepton masses break this degeneracy, there must be

radiative corrections[6] which may or may not be consistent with the actual phenomenological solutions desired, especially so if a value of 10^{-10} eV^2 for Δm_{12}^2 is to be maintained.

In this paper, a detailed model of lepton mass matrices is presented, based on the permutation symmetry S_3. It is the outgrowth of a previous proposal[7] which shows how Δm_{12}^2 of order 10^{-10} eV^2 for solar neutrino vacuum oscillations can be obtained as a second-order perturbation of a two-fold degenerate neutrino mass matrix, resulting in a successful formula relating atmospheric and solar neutrino oscillations. In Sec. 2 the model is described and it is shown how the breaking of S_3 together with the electroweak gauge symmetry allows the charged-lepton masses to be all different while maintaining a two-fold degeneracy in the meutrino mass matrix $\mathcal{M}_\nu$ at tree level. In Sec. 3 radiative corrections to $\mathcal{M}_\nu$ are derived in terms of the parameters of the charged-lepton mass matrix $\mathcal{M}_l$. The latter is chosen such that neutrinoless double beta decay[8] is absent at tree level. In Sec. 4 the resulting phenomenon of lepton flavor nonconservation beyond that of neutrino oscillations is discussed in the context of other processes involving charged leptons. Finally in Sec. 5 there are some concluding remarks.

2 Lepton mass matrices under S_3

Let the three families of leptons be denoted by $(\nu_i, l_i)_L$ and l^c_{iL}, $i = 1, 2, 3$. In this convention, $l_{iL} l^c_{jL}$ is a Dirac mass term for the charged leptons (instead of the usual $\bar{l}_{iL} l_{jR}$) and $\nu_i \nu_j$ is a Majorana mass term for the neutrinos. Consider the discrete permutation symmetry S_3. Its irreducible representations are $\underline{2}$, $\underline{1}$, and $\underline{1}'$, with the following multiplication rules: $\underline{2} \times \underline{2} = \underline{2} + \underline{1} + \underline{1}'$ and $\underline{1}' \times \underline{1}' = \underline{1}$. Under S_3, let

$$\left[\begin{pmatrix} \nu_1 \\ l_1 \end{pmatrix}_L, \begin{pmatrix} \nu_2 \\ l_2 \end{pmatrix}_L \right] \sim 2, \quad \begin{pmatrix} \nu_3 \\ l_3 \end{pmatrix}_L \sim 1, \quad [l^c_{1L}, l^c_{2L}] \sim 2, \quad l^c_{3L} \sim 1, \tag{6}$$

The Higgs sector of this model consists of three doublets $\Phi_i = (\phi_i^0, \phi_i^-), i = 1, 2, 3$, and one triplet $\xi = (\xi^{++}, \xi^+, \xi^0)$. Under S_3, let

$$(\Phi_1, \Phi_2) \sim 2, \quad \Phi_3 \sim 1, \quad \xi \sim 1. \tag{7}$$

Neutrinos couple to ξ according to

$$f_{ij}[\xi^0 \nu_i \nu_j + \xi^+(\nu_i l_j + l_i \nu_j)/\sqrt{2} + \xi^{++} l_i l_j] + h.c., \tag{8}$$

where f_{ij} is restricted by S_3 to have the form

$$f = \begin{pmatrix} 0 & f_0 & 0 \\ f_0 & 0 & 0 \\ 0 & 0 & f_3 \end{pmatrix}. \tag{9}$$

As shown recently[9, 10], this is an equally natural way to obtain small Majorana neutrino masses as the canonical seesaw mechanism[11], because the vacuum expectation value $\langle \xi^0 \rangle = u$ is inversely proportional to m_ξ^2. Let $m_0 = 2f_0 u$ and $m_3 = 2f_3 u$, then the Majorana neutrino mass matrix spanning $\nu_{1,2,3}$ is given by

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_0 & 0 \\ m_0 & 0 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{10}$$

The eigenvalues of $\mathcal{M}_\nu$ are $-m_0$, m_0, and m_3. [A negative mass here means that the corresponding Majorana neutrino is odd under CP after a γ_5 rotation to remove the minus sign.] Hence there is an effective two-fold degeneracy in the $\nu_1 - \nu_2$ sector, and it corresponds to an additional global symmetry, i.e. $L_1 - L_2$, if the charged-lepton mass matrix $\mathcal{M}_l$ is diagonal in the same basis.

There are five Yukawa interaction terms of the charged leptons with the Higgs doublets which are invariant under S_3, i.e.

$$h_1[l_1 l_1^c \phi_1^0 + l_2 l_2^c \phi_2^0 - \nu_1 l_1^c \phi_1^- - \nu_2 l_2^c \phi_2^-]$$

$$
\begin{aligned}
&+ \quad h_2[(l_1 l_2^c + l_2 l_1^c)\phi_3^0 - (\nu_1 l_2^c + \nu_2 l_1^c)\phi_3^-] \\
&+ \quad h_3[(l_1 \phi_2^0 + l_2 \phi_1^0) l_3^c - (\nu_1 \phi_2^- + \nu_2 \phi_1^-) l_3^c] \\
&+ \quad h_4[l_3(l_1^c \phi_2^0 + l_2^c \phi_1^0) - \nu_3(l_1^c \phi_2^- + l_2^c \phi_1^-)] \\
&+ \quad h_5[l_3 l_3^c \phi_3^0 - \nu_3 l_3^c \phi_3^-] + h.c. \qquad (11)
\end{aligned}
$$

As $\phi^0_{1,2,3}$ acquire vacuum expectation values $v_{1,2,3}$, the 3×3 mass matrix linking $l_{1,2,3}$ to $l^c_{1,2,3}$ is given by

$$
\mathcal{M}_l = \begin{bmatrix} h_1 v_1 & h_2 v_3 & h_3 v_2 \\ h_2 v_3 & h_1 v_2 & h_3 v_1 \\ h_4 v_2 & h_4 v_1 & h_5 v_3 \end{bmatrix}. \qquad (12)
$$

The scalar potential of this model is assumed to respect S_3 only in its dimension-four terms, i.e. S_3 is broken softly by its dimension-two and dimension-three terms:

$$
\begin{aligned}
&\sum_{i,j=1}^{3} m_{ij}^2 (\bar{\phi}_i^0 \phi_j^0 + \phi_i^+ \phi_j^-) + m_\xi^2 (\xi^{--}\xi^{++} + \xi^- \xi^+ + \bar{\xi}^0 \xi^0) \\
&+ \sum_{i,j=1}^{3} \mu_{ij} \left[\xi^{++} \phi_i^- \phi_j^- + \frac{1}{\sqrt{2}} \xi^+ (\phi_i^0 \phi_j^- + \phi_i^- \phi_j^0) + \xi^0 \phi^0 \phi^0 \right] + h.c. \qquad (13)
\end{aligned}
$$

This allows $\mathcal{M}_l$ to break S_3 with $v_2 \neq v_1$. In fact, the limits $h_2 = 0$ and $v_2 = 0$ are assumed here so that $\mathcal{M}_l$ becomes of the form

$$
\mathcal{M}_l = \begin{pmatrix} m_e & 0 & 0 \\ 0 & 0 & a \\ 0 & b & d \end{pmatrix}, \qquad (14)
$$

with l_1 identified as the electron, and

$$
m^2_{\tau,\mu} = \frac{1}{2}(d^2 + a^2 + b^2) \pm \frac{1}{2}\sqrt{d^2 + (a+b)^2}\sqrt{d^2 + (a-b)^2}. \qquad (15)
$$

From Eqs. (10) and (14), the eigenstates of $\mathcal{M}_\nu$ are easily read off:

$$
\nu_1 = \frac{1}{\sqrt{2}}(\nu_e - c\nu_\mu - s\nu_\tau), \quad \nu_2 = \frac{1}{\sqrt{2}}(\nu_e + c\nu_\mu + s\nu_\tau), \quad \nu_3 = -s\nu_\mu + c\nu_\tau, \qquad (16)
$$

where $c = \cos\theta_L$ and $s = \sin\theta_L$ are determined by the $\mu_L - \tau_L$ sector of Eq. (14). This means that ν_e mixes maximally with $c\nu_\mu + s\nu_\tau$, i.e. $\theta = \pi/4$ in Eq. (3). If $c = s = 1/\sqrt{2}$ is

also assumed in the above (corresponding to $a^2 = b^2 + d^2$), the so-called bimaximal form of neutrino oscillations is obtained. In that case,

$$a^2 = \frac{1}{2}(m_\tau^2 + m_\mu^2), \quad b^2 = \frac{2m_\tau^2 m_\mu^2}{m_\tau^2 + m_\mu^2}, \quad d^2 = \frac{(m_\tau^2 - m_\mu^2)^2}{2(m_\tau^2 + m_\mu^2)}. \tag{17}$$

3 Radiative corrections to neutrino mass degeneracy

To discuss radiative corrections to $\mathcal{M}_\nu$, consider first the case of keeping only one Higgs doublet Φ and one one Higgs triplet ξ. In this scenario, S_3 is explicitly broken by the Yukawa interactions of the charged leptons. Consequently, there is an arbitrariness in choosing the mass scale at which S_3 is assumed to be exact. The most natural choice in the present context is of course m_ξ, hence there are two contributions to the radiatively corrected $\mathcal{M}_\nu$. One is a finite correction to the mass matrix, as shown in Figure 1. The other is a renormalization of the coupling matrix from the shift in mass scale from m_ξ to m_W. This was the specific case presented in Ref. [7].

Here the situation is different in two ways. First, S_3 is a good symmetry as far as the Yukawa couplings are concerned. Second, it is softly broken only at the electroweak energy scale. Hence there is no S_3 breaking contribution from the renormalization of the coupling matrix. As for the finite correction to the mass matrix, because of the approximations $h_2 = 0$ and $v_2 = 0$, there are now only two contributions: $\nu_1\nu_3\bar{\phi}_1^0\bar{\phi}_3^0$ and $\nu_3\nu_3\bar{\phi}_3^0\bar{\phi}_3^0$, as shown in Figure 1. Assuming that μ_{33} dominates among all the μ_{ij}'s in Eq. (13) so that[9] $u \simeq -\mu_{33}v_3^2/m_\xi^2$, the mass matrix $\mathcal{M}_\nu$ is now corrected to read

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & m_0 & adIm_0 \\ m_0 & 0 & 0 \\ adIm_0 & 0 & (1+2d^2I)m_3 \end{pmatrix}. \tag{18}$$

The integral I is given by[7]

$$I = \frac{G_F}{4\pi^2\sqrt{2}\sin^2\beta}\ln\frac{m_\xi^2}{m_W^2}, \tag{19}$$

where $\sin^2\beta = v_3^2/(v_1^2+v_3^2)$. The two-fold degeneracy of the $\nu_1 - \nu_2$ sector is then lifted, with the following mass eigenvalues:

$$-m_0 - \frac{a^2d^2I^2m_0^2}{2(m_0+m_3)}, \quad m_0 + \frac{a^2d^2I^2m_0^2}{2(m_0-m_3)}, \tag{20}$$

where $adI << (m_0-m_3)/(m_0+m_3)$ has been used, being justified numerically. Hence their mass-squared difference is

$$\Delta m^2 \simeq a^2d^2I^2m_0^3\left[\frac{1}{m_0-m_3} - \frac{1}{m_0+m_3}\right] \simeq \frac{2a^2d^2I^2m_\nu^4}{m_0^2-m_3^2}, \tag{21}$$

where $m_\nu \simeq m_0 \simeq m_3$ has been used. Identifying this with solar neutrino vacuum oscillations then yields

$$\frac{(\Delta m^2)_{sol}(\Delta m^2)_{atm}}{m_\nu^4} = 2a^2d^2I^2 = \frac{2.16\times10^{-13}}{\sin^4\beta}\left(\ln\frac{m_\xi^2}{m_W^2}\right)^2, \tag{22}$$

where $a = 1259$ MeV and $d = 1250$ MeV from Eq. (17) have been used. In the above, bimaximal mixing (i.e. $\sin^2 2\theta_{sol} = \sin^2 2\theta_{atm} = 1$) has been assumed. For $(\Delta m^2)_{sol} \sim 4\times10^{-10}$ eV^2 in the case of vacuum oscillations and $(\Delta m^2)_{atm} \sim 4\times10^{-3}$ eV^2, this would require

$$\frac{m_\nu^4}{\sin^4\beta}\left(\ln\frac{m_\xi^2}{m_W^2}\right)^2 \sim 7.4 \text{ eV}^4, \tag{23}$$

which gives the bound $m_\nu < 0.6$ eV for $m_\xi > 1$ TeV and $\sin^2\beta < 0.7$. It is interesting to note that this same numerical limit in the case of three nearly mass-degenerate neutrinos was recently obtained[12] from the consideration of cosmic structure formation in the light of the latest astronomical observations.

The choice of Eq. (14) in conjunction with Eq. (10) means that neutrinoless double beta decay[8] is absent to lowest order. It also eliminates any one-loop correction to the diagonal entries of $\mathcal{M}_\nu$ in the $\nu_1 - \nu_2$ sector. This allows the mass splitting to be quadratic (as opposed to linear) in I as shown in Eq. (20), which is crucial for obtaining the very small phenomenological value of $(\Delta m^2)_{sol}$ for vacuum oscillations.

4 Lepton flavor nonconservation

Both lepton number and lepton flavor are not conserved in this model. Whereas lepton number nonconservation originates from the heavy Higgs triplet ξ and manifests itself at low energy only through the very small Majorana neutrino masses, lepton flavor nonconservation originates from the much less heavy Higgs doublets which are presumably in the 100 GeV mass range. On the other hand, the h_i's of Eq. (11) are suppressed relative to the gauge couplings because they are related to $\mathcal{M}_l$ as shown in Eqs. (12) and (14).

Using Eqs. (14) and (17), it is easily shown that

$$\begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & -1/\sqrt{2} \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix}, \tag{24}$$

and

$$\begin{pmatrix} l_1^c \\ l_2^c \\ l_3^c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_R & \sin\theta_R \\ 0 & -\sin\theta_R & \cos\theta_R \end{pmatrix} \begin{pmatrix} e^c \\ \mu^c \\ \tau^c \end{pmatrix}, \tag{25}$$

where

$$\tan\theta_R = \frac{m_\mu}{m_\tau}\left(\frac{m_\tau^2 - m_\mu^2}{m_\tau^2 + m_\mu^2}\right). \tag{26}$$

The interactions of $\phi^0_{1,2,3}$ are then given by

$$\begin{aligned}
&\phi_1^0\left[h_1 e e^c + \frac{-h_3 s_R + h_4 c_R}{\sqrt{2}}\mu\mu^c + \frac{-h_3 c_R + h_4 s_R}{\sqrt{2}}\tau\tau^c + \frac{h_3 c_R + h_4 s_R}{\sqrt{2}}\mu\tau^c + \frac{h_3 s_R + h_4 c_R}{\sqrt{2}}\tau\mu^c\right] + \\
&\phi_2^0\left[\frac{h_1 c_R}{\sqrt{2}}\mu\mu^c - \frac{h_1 s_R}{\sqrt{2}}\tau\tau^c - h_3 s_R e\mu^c + \frac{h_4}{\sqrt{2}}\mu e^c + h_3 c_R e\tau^c + \frac{h_4}{\sqrt{2}}\tau e^c + \frac{h_1 s_R}{\sqrt{2}}\mu\tau^c - \frac{h_1 c_R}{\sqrt{2}}\tau\mu^c\right] + \\
&\phi_3^0\left[-\frac{h_5 s_R}{\sqrt{2}}\mu\mu^c + \frac{h_5 c_R}{\sqrt{2}}\tau\tau^c + h_1 c_R e\mu^c + \frac{h_1}{\sqrt{2}}\mu e^c + h_1 s_R e\tau^c - \frac{h_1}{\sqrt{2}}\tau e^c + \frac{h_5 c_R}{\sqrt{2}}\mu\tau^c - \frac{h_5 s_R}{\sqrt{2}}\tau\mu^c\right],
\end{aligned} \tag{27}$$

where $s_R = \sin\theta_R$ and $c_R = \cos\theta_R$. In the above,

$$h_1 = \frac{m_e}{v_1}, \quad h_3 \simeq \frac{m_\tau}{v_1\sqrt{2}}, \quad h_4 \simeq \frac{\sqrt{2}m_\mu}{v_1}, \quad h_5 \simeq \frac{m_\tau}{v_3\sqrt{2}}, \quad s_R \simeq \frac{m_\mu}{m_\tau}, \quad c_R \simeq 1. \tag{28}$$

Consequently, the most prominent rare decays are $\tau^- \to e^- e^- \mu^+$, $\tau^- \to e^- \mu^- \mu^+$, and $\tau^- \to \mu^- \mu^- \mu^+$, with branching fractions

$$B(\tau^- \to e^- e^- \mu^+) \simeq 2B(\tau^- \to e^- \mu^- \mu^+) \simeq \frac{m_\mu^2 m_\tau^2}{8 \cos^4 \beta m_{\phi_2^0}^4}, \tag{29}$$

and

$$B(\tau^- \to \mu^- \mu^- \mu^+) \simeq \frac{m_\mu^2 m_\tau^2}{16} \left(\frac{1}{\cos^2 \beta m_{\phi_1^0}^2} - \frac{1}{\sin^2 \beta m_{\phi_3^0}^2} \right)^2. \tag{30}$$

With the scalar masses of order 100 GeV, these branching fractions are of order 10^{-10}, much below the present experimental upper limits[13] which are of order 10^{-6}. Note that $\mu \to eee$ is suppressed even more strongly in this model because its amplitude is proportional to $m_e m_\mu$.

Whereas low-energy tests of this model are limited to neutrino masses and oscillations, dramatic effects are predicted at high energies. The production of $\phi_{1,2,3}^0$ at future colliders would yield very clear signals from decays such as $\phi_2^0 \to \tau^- e^+$ and $\phi_{1,3}^0 \to \tau^- \mu^+$.

5 Concluding remarks

To understand the present experimental data on atmospheric and solar neutrinos, a model of neutrino and charged-lepton mass matrices based on the permutation symmetry S_3 has been proposed. It has a two-fold degeneracy in the neutrino mass matrix which is broken radiatively, and allows for a very small mass splitting, suitable for solar neutrino vacuum oscillations, as given by Eq. (22). The S_3 symmetry is maintained in the scalar sector by three Higgs doublets which determine the charged-lepton mass matrix, whereas the neutrinos obtain naturally small Majorana masses from their couplings to a heavy Higgs triplet. Lepton flavor nonconservation at low energy is suppressed by the small charged-lepton masses. This model may be tested at high energy with the production and decay of its scalar doublets.

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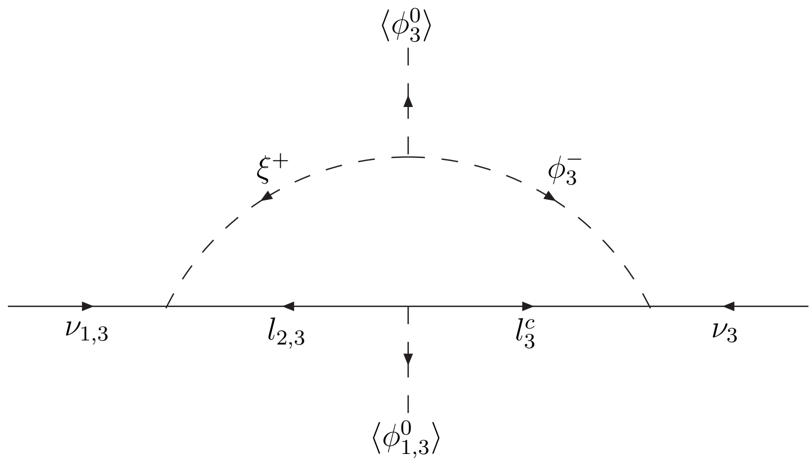

Fig. 1. One-loop radiative breaking of neutrino mass degeneracy.